Sistemas de Eq diferenciais

$$\begin{cases} \dfrac{dx}{dt} = f(\bar{x}, \bar{y}) = 0 \\[2ex] \dfrac{dy}{dt} = g(\bar{x}, \bar{y}) = 0 \end{cases}$$

$$f(\bar{x}, \bar{y}) = 0$$
$$g(\bar{x}, \bar{y}) = 0$$

$$x(t) = \underbrace{\bar{x} + x'}(t) \longrightarrow x'(t) = x(t) - \bar{x}$$

$$\frac{dx'}{dt} = \frac{dx}{dt}$$

$$y(t) = \underbrace{\bar{y} + y'}(t) \longrightarrow y'(t) = y(t) - \bar{y}$$

$$\frac{dy'}{dt} = \frac{dy}{dt}$$

$$\frac{dx}{dt} = f(x, y)$$

$$\frac{dx}{dt} = f(\bar{x} + x', \bar{y} + y') \approx f(\bar{x}, \bar{y}) + \frac{df}{dx}(\bar{x}, \bar{y})\, x' + \frac{df}{dy}(\bar{x}, \bar{y})\, y'$$

$$\frac{dy}{dt} = g(\bar{x}+x', \bar{y}+y') \approx g(\bar{x},\bar{y}) + \frac{dg}{dx}(\bar{x},\bar{y})\,x' + \frac{dg}{dy}(\bar{x},\bar{y})\,y'$$

$$\begin{cases} \underbrace{\dfrac{dx'}{dt}} = \dfrac{dx}{dt} = \cancelto{0}{f(\bar{x},\bar{y})} + \underbrace{\dfrac{df}{dx}(\bar{x},\bar{y})\,x' + \dfrac{df}{dy}(\bar{x},\bar{y})\,y'} \\[2ex] \underbrace{\dfrac{dy'}{dt}} = \dfrac{dy}{dt} = \cancelto{0}{g(\bar{x},\bar{y})} + \underbrace{\dfrac{dg}{dx}(\bar{x},\bar{y})\,x' + \dfrac{dg}{dy}(\bar{x},\bar{y})\,y'} \end{cases}$$

$$\begin{cases} \dfrac{dx'}{dt} = \underbrace{\dfrac{df}{dx}(\bar{x},\bar{y})}\,x' + \underbrace{\dfrac{df}{dy}(\bar{x},\bar{y})}\,y' \\[2ex] \dfrac{dy'}{dt} = \underbrace{\dfrac{dg}{dx}(\bar{x},\bar{y})}\,x' + \underbrace{\dfrac{dg}{dy}(\bar{x},\bar{y})}\,y' \end{cases}$$

$$\vec{v} = \begin{pmatrix} x' \\ y' \end{pmatrix} \qquad \frac{d\vec{v}}{dt} = \begin{pmatrix} \frac{dx'}{dt} \\ \frac{dy'}{dt} \end{pmatrix}$$

$$\frac{dx'}{dt} = \sim\sim x' + \sim\sim y'$$

$$\frac{dy'}{dt} = \sim\sim x' + \sim\sim y'$$

$$\frac{d\vec{v}}{dt} = A\vec{v}$$

$$\underbrace{\left.\begin{pmatrix} \frac{df}{dx} & \frac{df}{dy} \\ \frac{dg}{dx} & \frac{dg}{dy} \end{pmatrix}\right|_{\bar{x},\bar{y}}}$$

Jacobiana comunitária

$$A = \begin{pmatrix} \frac{df}{dx} & \frac{df}{dy} \\ \frac{dg}{dx} & \frac{dg}{dy} \end{pmatrix}\Bigg|_{\bar{x},\bar{y}}$$

$\lambda_+ \quad v_+$

$\lambda_- \quad v_-$

$$\vec{v} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\frac{d\vec{v}}{dt} = A\vec{v}$$

$$\frac{dN(t)}{dt} = kN(t) \rightsquigarrow \underbrace{N(t) = Ce^{kt}}$$

$$\vec{v}(t) = C_+ v_+ e^{\lambda_+ t} + C_- v_- e^{\lambda_- t}$$

$\downarrow$

vetor de perturbações

- λ_+ e $\lambda_- < 0$

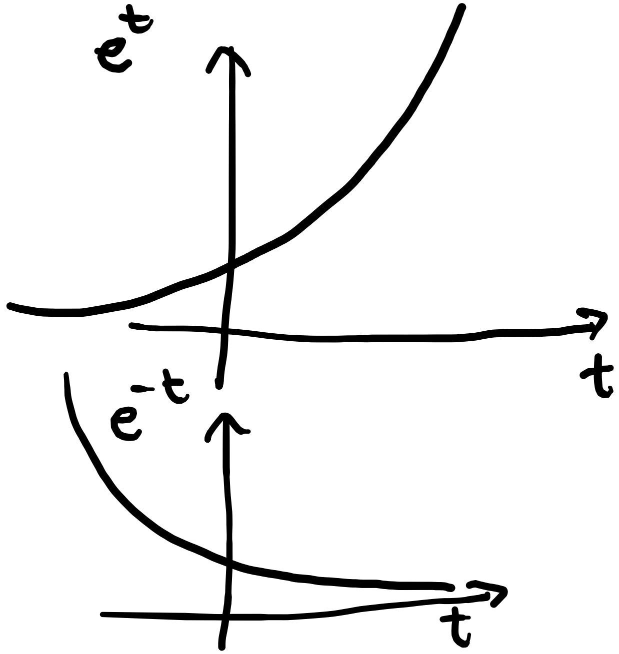
  → pert. diminuem

  ⇝ estável

- λ_+ ou $\lambda_- > 0$

  → perturbações crescem

  ⇝ instável



Estabilidade de competição:

$$\frac{dN_1}{dt} = f(N_1,N_2) = r_1 N_1 - \frac{r_1 N_1^2}{K_1} - \frac{r_1 \alpha_{12}}{K_1} N_1 N_2 = \frac{r_1 N_1}{K_1}\left(K_1 - N_1 - \alpha_{12} N_2\right)$$

$$\frac{dN_2}{dt} = g(N_1,N_2) = r_2 N_2 - \frac{r_2 N_2^2}{K_2} - \frac{r_2 \alpha_{21}}{K_2} N_1 N_2 = \frac{r_2 N_2}{K_2}\left(K_2 - N_2 - \alpha_{21} N_1\right)$$

Equilíbrio: $f(N_1,N_2)=0$ e $g(N_1,N_2)=0$

$$\frac{r_1 N_1}{K_1}\left(K_1 - N_1 - \alpha_{12} N_2\right) = 0$$

$$\frac{r_2 N_2}{K_2}\left(K_2 - N_2 - \alpha_{21} N_1\right) = 0$$

$\bar{N}_1 = 0$ //

$K_1 - \bar{N}_1 - \alpha_{12}\bar{N}_2 = 0$

$\bar{N}_1 = K_1 - \alpha_{12}\bar{N}_2$ //

$\bar{N}_2 = 0$ //

$K_2 - \bar{N}_2 - \alpha_{21}\bar{N}_1 = 0$

$\bar{N}_2 = K_2 - \alpha_{21}\bar{N}_1$ //

1) $\bar{N}_1 = 0 \quad \bar{N}_2 = 0$

2) $\bar{N}_1 = 0 \quad \bar{N}_2 = K_2 - \alpha_{21}\bar{N}_1 = K_2$

3) $\bar{N}_1 = K_1 - \alpha_{12}\bar{N}_2 = K_1 \quad \bar{N}_2 = 0$

4) $\bar{N}_1 = K_1 - \alpha_{12}(K_2 - \alpha_{21}\bar{N}_1)$

$\bar{N}_1 = K_1 - \alpha_{12}K_2 - \underline{\alpha_{12}\alpha_{21}\bar{N}_1}$

$\bar{N}_1 - \alpha_{12}\alpha_{21}\bar{N}_1 = K_1 - \alpha_{12}K_2$

$\bar{N}_1(1 - \alpha_{12}\alpha_{21}) = K_1 - \alpha_{12}K_2$

$$\bar{N}_1 = \frac{K_1 - \alpha_{12}K_2}{1 - \alpha_{12}\alpha_{21}}$$

$\bar{N}_2 = K_2 - \alpha_{21}\bar{N}_1$

$$\bar{N}_2 = \frac{K_2 - \alpha_{21}K_1}{1 - \alpha_{21}\alpha_{12}}$$

Estabilidade:

$$f(N_1,N_2) = r_1 N_1 - \frac{r_1}{K_1} N_1^2 - \frac{r_1}{K_1}\alpha_{12} N_1 N_2$$

$$g(N_1 N_2) = r_2 N_2 - \frac{r_2}{K_2} N_2^2 - \frac{r_2}{K_2}\alpha_{21} N_1 N_2$$

Jacobiana: $\begin{pmatrix} \frac{df}{dN_1} & \frac{df}{dN_2} \\ dg/dN_1 & dg/dN_2 \end{pmatrix}$

$$J = \begin{pmatrix} r_1 - 2\frac{r_1}{K_1}N_1 - \frac{r_1}{K_1}\alpha_{12}N_2 & -\frac{r_1}{K_1}\alpha_{12}N_1 \\ -\frac{r_2}{K_2}\alpha_{21}N_2 & r_2 - \frac{2r_2}{K_2}N_2 - \frac{r_2}{K_2}\alpha_{21}N_1 \end{pmatrix}$$

$r_1 = 0{,}9 \quad \alpha_{12} = 0.6 \quad k_1 = k_2 = 500$

$r_2 = 0{,}5 \quad \alpha_{21} = 0{,}7$

$J\big|_{\bar{N}_1, \bar{N}_2}$

1) $\bar{N}_1 = 0 \quad \bar{N}_2 = 0$

$$\underbrace{J = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}}$$

$$\det \begin{pmatrix} r_1 - \lambda & 0 \\ 0 & r_2 - \lambda \end{pmatrix} = 0 \qquad (r_1 - \lambda)(r_2 - \lambda) = 0 \quad \begin{cases} \lambda_+ = 0.9 \\ \lambda_- = 0.5 \end{cases}$$

$\Rightarrow (0,0)$ instável

$\bar{N}_1 = 0 \qquad \bar{N}_2 = k_2 = 500$

$r_1 = 0.9 \quad r_2 = 0.5 \qquad k_1 = k_2 = 500$

$\alpha_{12} = 0.6 \quad \alpha_{21} = 0.7$

$$\begin{pmatrix} r_1 - \frac{2r_1}{k_1} N_1 - \frac{r_1 \alpha_{12}}{k_1} N_2 & -\frac{r_1 \alpha_{12} N_1}{k_1} \\ -\frac{r_2 \alpha_{21}}{k_2} N_2 & r_2 - \frac{2r_2}{k_2} N_2 - \frac{r_2 \alpha_{21}}{k_2} N_1 \end{pmatrix}$$

$$J \begin{pmatrix} 0{,}9 - \frac{0{,}9 \cdot 0{,}6}{500} 500 & 0 \\ -\frac{0.5 \cdot 07}{\cancel{500}} \cdot \cancel{500} & 0{,}5 - \frac{2{,}0{,}5}{\cancel{500}} \cdot \cancel{500} \end{pmatrix} = \begin{pmatrix} 0.9 - 0{,}54 & 0 \\ -0.35 & +0{,}5 - 1 \end{pmatrix}$$

$$\begin{pmatrix} 0{,}36 & 0 \\ -0.35 & -0{,}5 \end{pmatrix}$$

$$\det\begin{pmatrix} 0{,}36-\lambda & 0 \\ -0{,}35 & -0{,}5-\lambda \end{pmatrix} = 0$$

$$(0{,}36-\lambda)(-0{,}5-\lambda) - 0 = 0 \quad \begin{cases} \lambda_+ = 0.36 \\ \lambda_- = -0.5 \end{cases}$$

$\bar{N}_1 = 0 \quad N_2 = k_2$ é instável

$\rightarrow N_1 = 500 \quad N_2 = 0$

$\rightarrow N_1$ e $N_2 \neq 0$