$$\begin{cases} P_{n+1} = \lambda e^{-aP_n} H_n \\ H_{n+1} = c H_n \left(1 - e^{-aP_n}\right) \end{cases}$$

Nicholson-Bailey

$$\bar{H} = \bar{P} = 0$$

$$\bar{H} = \ln \lambda \left(\frac{\lambda}{ac(\lambda - 1)} \right)$$

$$\bar{P} = \frac{\ln \lambda}{a}$$

$$\begin{cases} H_{n+1} = \lambda H_n e^{-aP_n} = f(H_n, P_n) \\ P_{n+1} = c\left(1 - e^{-aP_n}\right) H_n = g(H_n, P_n) \end{cases}$$

Matriz jacobiana

$$J = \begin{pmatrix} \frac{df}{dH} & \frac{df}{dP} \\ \frac{dg}{dH} & \frac{dg}{dP} \end{pmatrix} = \begin{pmatrix} \lambda e^{-aP} & -a\underbrace{\lambda H e^{-aP}}_{H} \\ c\left(1 - e^{-aP}\right) & cH\left(a e^{-aP}\right) \end{pmatrix} = \begin{pmatrix} \lambda e^{-aP} & -aH \\ c\left(1 - e^{-aP}\right) & \frac{ac}{\lambda} H \end{pmatrix}$$

$$\Rightarrow (H_0, P_0) = \underline{(0,0)}$$

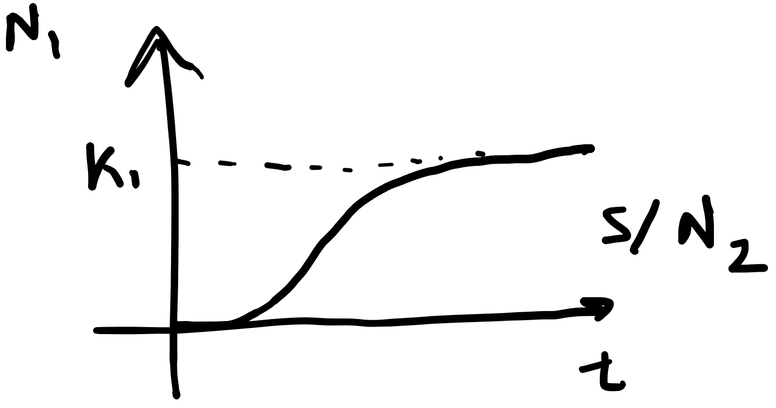
$$J\Big|_{(H_0,P_0)} = \begin{pmatrix} \lambda & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{matrix} \Gamma_1 = \lambda \\ \Gamma_2 = 0 \end{matrix}$$

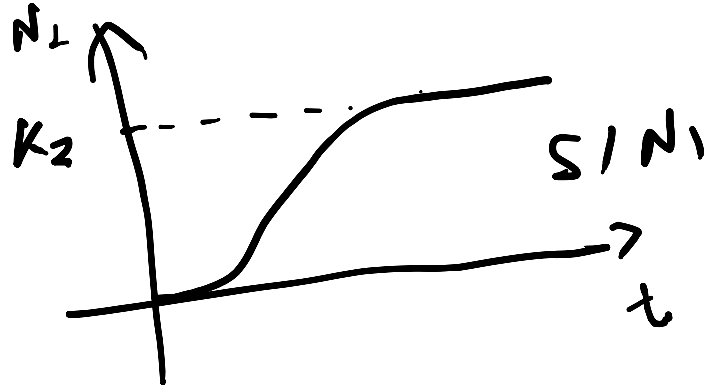
estável se $|\lambda| < 1$

instável se $|\lambda| > 1$

Modelo de competição

$$\begin{cases} \dfrac{dN_1}{dt} = \dfrac{r_1 N_1}{K_1}\left(K_1 - N_1 - \alpha_{12} N_2\right) \\[2ex] \dfrac{dN_2}{dt} = \dfrac{r_2 N_2}{K_2}\left(K_2 - N_2 - \alpha_{21} N_1\right) \end{cases}$$





Pontos de Eq.: $\dfrac{dN_1}{dt} = \dfrac{dN_2}{dt} = 0$

* $\bar{N}_1 = \bar{N}_2 = 0$

$\bar{N}_1 = 0 \quad \bar{N}_2 = K_2$

$\bar{N}_1 = K_1 \quad \bar{N}_2 = 0$

$$\begin{cases} N_1 = \dfrac{K_1 - \alpha_{12} K_2}{1 - \alpha_{12}\alpha_{21}} \\[2ex] N_2 = \dfrac{K_2 - \alpha_{21} K_1}{1 - \alpha_{21}\alpha_{12}} \end{cases}$$

$\bar{N}_1 = 0 \qquad \bar{N}_2 = k_2 = 500$

$r_1 = 0.9 \quad r_2 = 0.5 \qquad k_1 = k_2 = 500$

$\alpha_{12} = 0{,}6 \quad \alpha_{21} = 0{,}7$

$$\begin{pmatrix} r_1 - \frac{2r_1}{k_1} N_1 - \frac{r_1 \alpha_{12}}{k_1} N_2 & -\frac{r_1 \alpha_{12} N_1}{k_1} \\ -\frac{r_2 \alpha_{21}}{k_2} N_2 & r_2 - \frac{2r_2}{k_2} N_2 - \frac{r_2 \alpha_{21}}{k_2} N_1 \end{pmatrix}$$

$$J \begin{pmatrix} 0{,}9 - \frac{0{,}9 \cdot 0{,}6}{500} 500 & 0 \\ -\frac{0.5 \cdot 07}{\cancel{500}} \cdot \cancel{500} & 0{,}5 - \frac{2{,}0{,}5}{\cancel{500}} \cdot \cancel{500} \end{pmatrix} = \begin{pmatrix} 0.9 - 0{,}54 & 0 \\ -0.35 & +0{,}5 - 1 \end{pmatrix}$$

$$\begin{pmatrix} 0{,}36 & 0 \\ -0.35 & -0{,}5 \end{pmatrix}$$

$\lambda_1 = \text{real} = c$

$\lambda_2 = \text{comp} = a + bi$

$\lambda_3 = \text{''} = a - bi$

$|\lambda_2| = |\lambda_3| = \rho = \sqrt{a^2 + b^2}$

$\lambda_1 = \lambda_1 = c = \sqrt{c^2 + 0^2}$

''

c

ρ q c

$$N_1 = N_2 = 0$$

$$J\Big|_{0,0} = \underbrace{\begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}}_{\text{autovalores da } J}$$

$$A\vec{v} = \lambda \vec{v}$$

$$A\vec{v} - \lambda \vec{v} = 0$$

$$\underbrace{(A - \lambda I)}\,\vec{v} = 0$$

$$\begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} r_1 - \lambda & 0 \\ 0 & r_2 - \lambda \end{pmatrix}$$

det $= 0$ $\quad \underbrace{(r_1 - \lambda)}\underbrace{(r_2 - \lambda)} - 0 = 0$

$$\lambda = r_1 \qquad \lambda = r_2$$

$$\begin{pmatrix} \sim & \sim \\ 0 & \sim \end{pmatrix}$$

$$\lambda = \frac{-0{,}87}{2} \underbrace{\pm \frac{1}{2}}_{\sim} \underbrace{\sqrt{0{,}87^2 - 4 \cdot 0{,}089}}_{< 0{,}87}$$

λ_1 e $\lambda_2 < 0$

$$\det \begin{pmatrix} -0{,}62 - \lambda & -0{,}37 \\ -0{,}18 & -0{,}25 - \lambda \end{pmatrix} = 0$$

$$(-0{,}62 - \lambda)(-0{,}25 - \lambda) - 0{,}066 = 0$$

$$0.155 + \underline{0{,}62\lambda} + \underline{0.25\lambda} + \lambda^2 - 0{,}066 = 0$$

$$\lambda^2 + 0.87\lambda + 0{,}089 = 0$$