

# Introdução à Teoria Jogos

$$p_{n+1} = \frac{p_n}{\bar{w}} (p_n w_{AA} + q_n w_{Aa})$$

$$\bar{w} = p^2 w_{AA} + 2pq w_{Aa} + q^2 w_{aa}$$



fenótipos / traits / estratégias

2 fenótipos

A e B

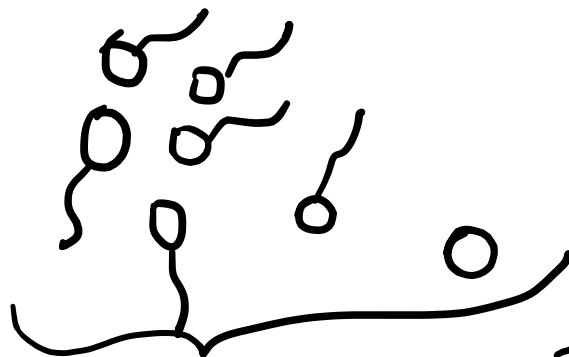
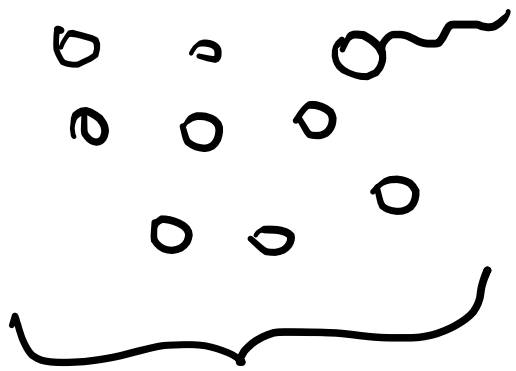


fitness A = 1.1



fitness B = 1

fit<sub>A</sub> > fit<sub>B</sub> → (A)



A ganha  $a$  na presença de outros A  
 A ganha  $b$  na presença de outros B

$x_A$   
 $x_B$   
 freq

Dependência na  
 frequência p/o  
 fitness do A

$$f_A = a x_A + b x_B$$

B ganha  $c$  na presença de A  
 B "  $d$  " " " B

$$f_B = c x_A + d x_B$$

$$x_A(t+1) = \frac{f_A x_A(t)}{\bar{w}} = \frac{f_A x_A}{x_A f_A + x_B f_B}$$

$$x_B(t+1) = \frac{f_B x_B(t)}{\bar{w}} = \frac{f_B x_B}{x_A f_A + x_B f_B}$$

$$x'_A = \frac{f_A}{\bar{w}} x_A$$

$$\boxed{x' = \frac{f_A}{\bar{w}} x}$$

$$\bar{w} = \underline{x_A f_A + x_B f_B}$$

$$= x_A (x_A a + x_B b) + x_B (x_A c + x_B d)$$

$$x_A + x_B = 1$$

$$x_A = x$$

$$x_B = 1 - x$$

$$p + q = 1$$

Equilibrio:

$$x' = \frac{f_A}{\bar{w}} x$$

$$x' = x = \bar{x}$$

$$\bar{x} = \frac{f_A}{\bar{w}} \bar{x} \Rightarrow$$

$$\bar{x} \bar{w} = f_A \bar{x}$$

$$f_A \bar{x} - \bar{w} \bar{x} = 0$$

$$\bar{x} (f_A - \bar{w}) = 0$$

$$\boxed{\bar{x} = 0}$$

$$\boxed{\bar{x} = 1}$$

$$(1 - \bar{x}) (f_A - f_B) = 0$$

$$f_A = f_B$$

$$\begin{aligned} f_A - \bar{w} &= \\ f_A - [x f_A + (1-x) f_B] &= \\ f_A - x f_A - (1-x) f_B &= \\ f_A (1-x) - (1-x) f_B &= \\ (1-x) (f_A - f_B) &= \end{aligned}$$

$$f_A = f_B$$

$$a\bar{x} + b(1-\bar{x}) = c\bar{x} + d(1-\bar{x})$$

$$a\bar{x} + b - b\bar{x} = c\bar{x} + d - d\bar{x}$$

$$a\bar{x} - b\bar{x} - c\bar{x} + d\bar{x} = d - b$$

$$\bar{x}(a+d-c-b) = d-b$$

$$\boxed{\bar{x} = \frac{d-b}{a+d-c-b}}$$

Estabilidade!

$$\left| \frac{df}{dx} \right|_{\bar{x}} < 1 \rightarrow \text{Estável}$$

$$> 1 \rightarrow \text{Instável}$$

$$x'_A = \frac{x_A f_A}{\bar{w}}$$

$$\vec{x} = \begin{pmatrix} x_A \\ x_B \end{pmatrix} = \begin{pmatrix} x \\ 1-x \end{pmatrix}$$

$$x'_B = \frac{x_B f_B}{\bar{w}}$$

$$\vec{f} = \begin{pmatrix} f_A \\ f_B \end{pmatrix} = \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{\text{Matriz de ganhos / payoffs}} \begin{pmatrix} x_A \\ x_B \end{pmatrix} = \begin{pmatrix} ax_A + bx_B \\ cx_A + dx_B \end{pmatrix}$$

Matriz de ganhos / payoffs