

2ª ordem linear (sistema de equações)

x e y x_n y_n

$$\begin{cases} x_{n+1} = a_{11} x_n + a_{12} y_n \\ y_{n+1} = a_{21} x_n + a_{22} y_n \end{cases}$$

$$V_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$V_{n+1} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$V_{n+1} = A V_n$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} a_{11} x_n + a_{12} y_n \\ a_{21} x_n + a_{22} y_n \end{pmatrix}$$

$$\begin{cases} x_{n+1} = a_{11} x_n + a_{12} y_n \rightarrow x_n = a_{11} x_{n-1} + a_{12} y_{n-1} \\ y_{n+1} = a_{21} x_n + a_{22} y_n \rightarrow y_n = a_{21} x_{n-1} + a_{22} y_{n-1} \end{cases}$$

$$x_{n+1} = a_{11} x_n + a_{12} y_n$$

$$x_{n+1} = a_{11} x_n + a_{12} (a_{21} x_{n-1} + a_{22} y_{n-1})$$

$$x_{n+1} = a_{11} x_n + a_{12} a_{21} x_{n-1} + a_{12} a_{22} y_{n-1}$$

$$x_{n+1} = a_{11} x_n + a_{12} a_{21} x_{n-1} + a_{22} (x_n - a_{11} x_{n-1})$$

$$x_{n+1} = \underline{a_{11}} x_n + \underline{a_{12} a_{21}} x_{n-1} + \underline{a_{22}} x_n - \underline{a_{22} a_{11}} x_{n-1}$$

$$x_{n+1} = (a_{11} + a_{22}) x_n + (a_{12} a_{21} - a_{22} a_{11}) x_{n-1} \quad 2^{\text{a}} \text{ order}$$

$$x_{n+1} = (a_{11} + a_{22}) x_n + (a_{12} a_{21} - a_{22} a_{11}) x_{n-1}$$

$$a_{11} + a_{22} = \beta$$

$$a_{12} a_{21} - a_{22} a_{11} = -\gamma \quad \leadsto \quad \gamma = a_{22} a_{11} - a_{12} a_{21}$$

$$x_{n+1} = \beta x_n - \gamma x_{n-1}$$

$$x_n = C_+ R_+^n + C_- R_-^n$$

x_0
 x_1
 a, σ, f
 $a_{11} a_{12}$
 $a_{21} a_{22}$

$$A = \begin{pmatrix} \underline{a_{11}} & a_{12} \\ a_{21} & \underline{a_{22}} \end{pmatrix}$$

$\beta =$ soma dos termos da diag. principal
 \rightarrow traço da matriz

$\gamma =$ determinante da matriz

Celhas de Fibonacci

1 casal de celhas

1 mês p/ os celhas virarem adultos

A partir do 2º mês esse casal começa a se reproduzir e dar origem a um casal de filhotes.

Quantos celhas terai em 2 anos?

MÊS	CASAL
0	-
1	J
2	A
3	AJ
4	AAJ
5	<u>AAAJJ</u>
6	<u>AAAAJJJ</u>
7	<u>AAAAAAJJJJ</u>

TOTAL

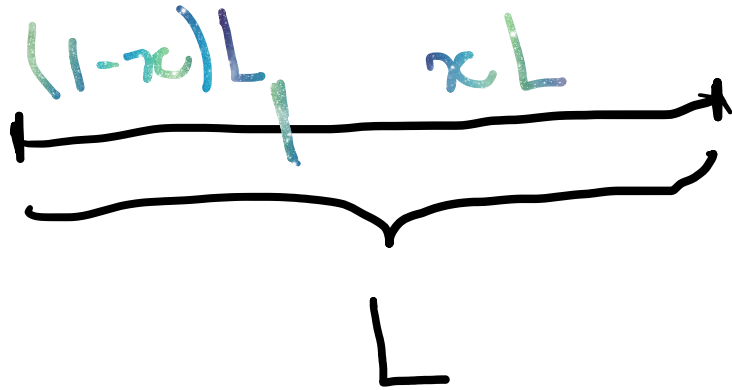
- 0
- 1
- 1
- 2
- 3
- 5
- 8
- 13

$$F_{n+1} = \underbrace{F_n}_{\text{Solter}} + \underbrace{F_{n-1}}_{\text{rep.}}$$

Eg geral

$\frac{F_{n+1}}{F_n} \rightarrow$ razão aurea

0.6180339



$$\frac{L}{xL} = \frac{Lx}{L(1-x)}$$

$$\frac{1}{x} = \frac{x}{1-x}$$

$$1-x = x^2$$

$$\leadsto x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{5}}{2} = \varphi^{0.618\dots}$$

Desafio:

Problema de Fibonacci

f_0

f_1

$$F_{n+1} = F_n + F_{n-1}$$

Eg geral

$$F_n = (\text{parâmetros e } n)$$

