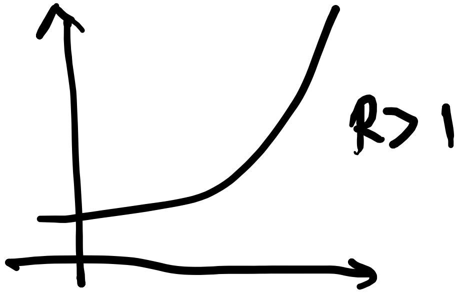
Comportamento qualitativo de soluções:

1ª ordem

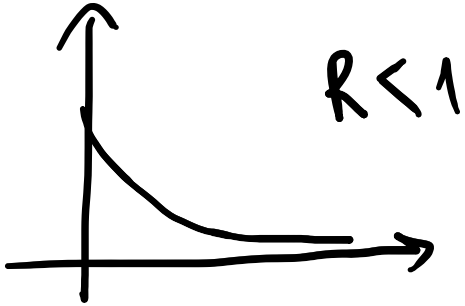
$$P_{n+1} = \beta P_n - \gamma P_{n-1}$$



$$R^2 - \beta R + \gamma = 0$$

$$R_\pm = \frac{\beta}{2} \pm \frac{1}{2}\sqrt{\underbrace{\beta^2 - 4\gamma}}$$

$$P_n = C_+ R_+^n + C_- R_-^n$$

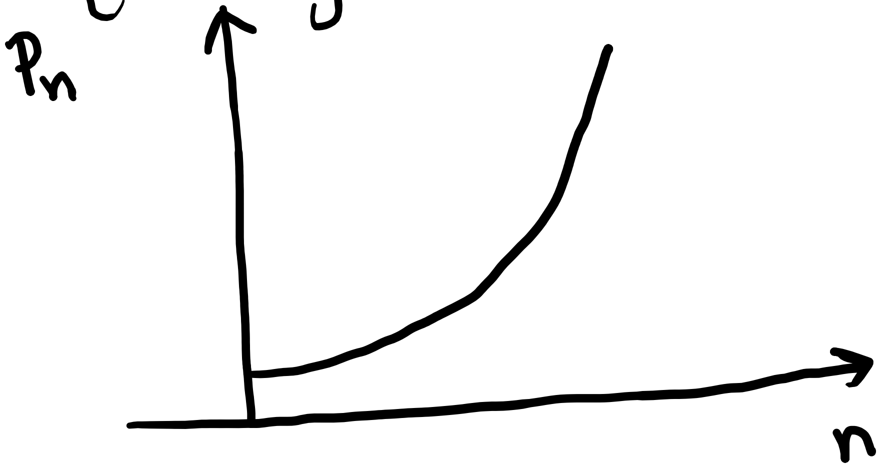


$$\beta^2 > 4\gamma$$

R com maior módulo (ou valor absoluto) é dito o R dominante

$\underline{\underline{3}}$ -2 } -0.5 $-\underline{\underline{2}}$ } $-\underline{\underline{5}}$ 0.5 }

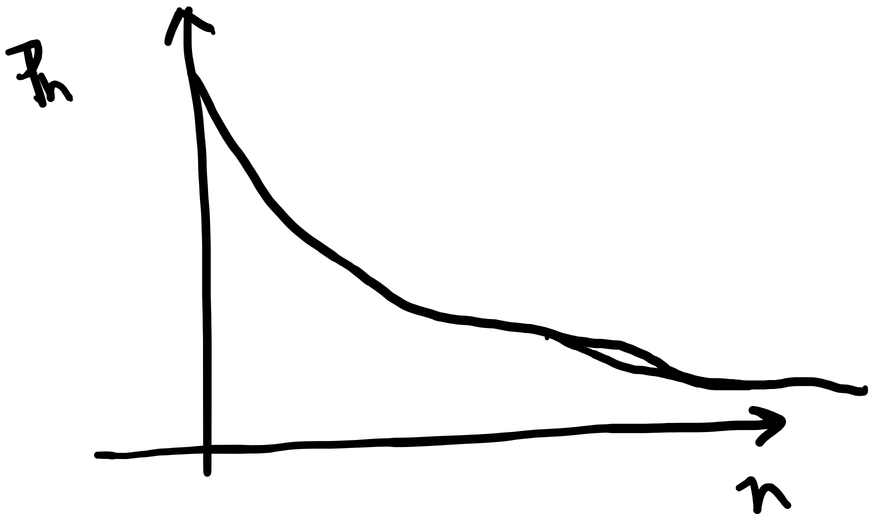
• 2 soluções cujo R de maior módulo é $R_+ > 1$



menor módulo

• 2 soluções e a solução R_+ é tq $0 < R_+ < 1$

Ex: 0.1 e 0.8

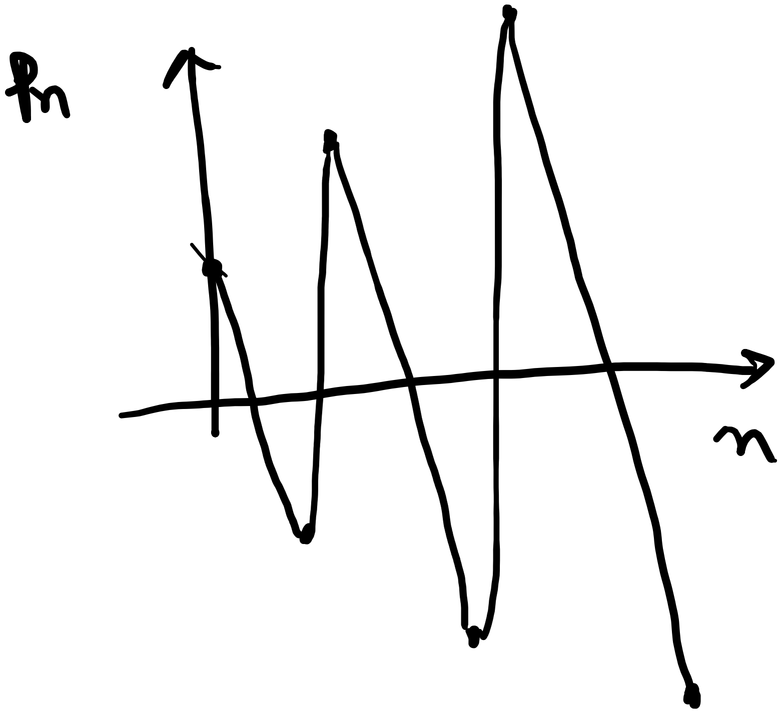


• 2 soluções e a de maior módulo R_+ é tq $\underbrace{R_+} < -1$

$$P_{n+1} = R_+ P_n$$

$R_+ = -2$

P_0



$$P_1 = -2P_0$$

$$P_2 = -2P_1 = -2\cdot(-2P_0) = 4P_0$$

$$P_3 = -2P_2 = -8P_0$$

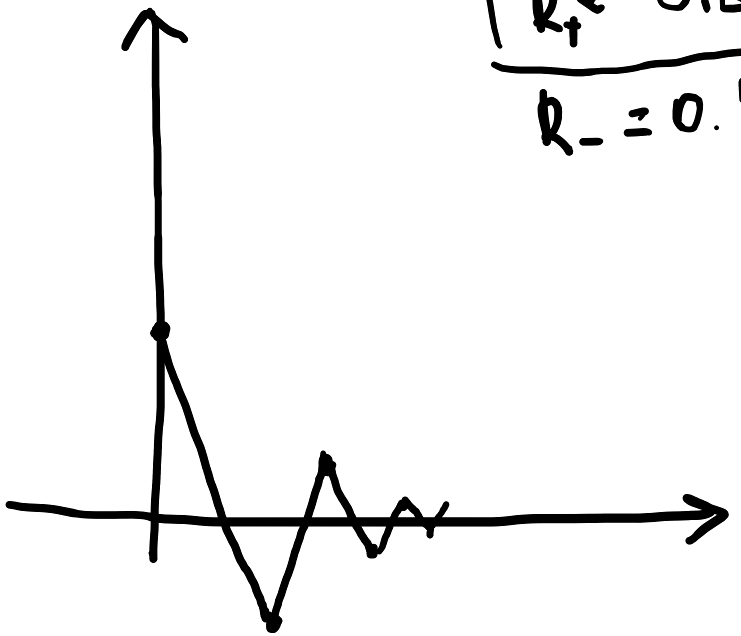
$$P_4 = -2P_3 = +16P_0$$

oscilações em zigzage crescentes

• 2 soluções e R_+ (dominante) é tq $-1 < R_+ < 0$ $R_+ = -0,5$

$\boxed{R_+ = -0,5}$

$R_- = 0.1$



$P_{n+1} = R_+ P_n$ P_0

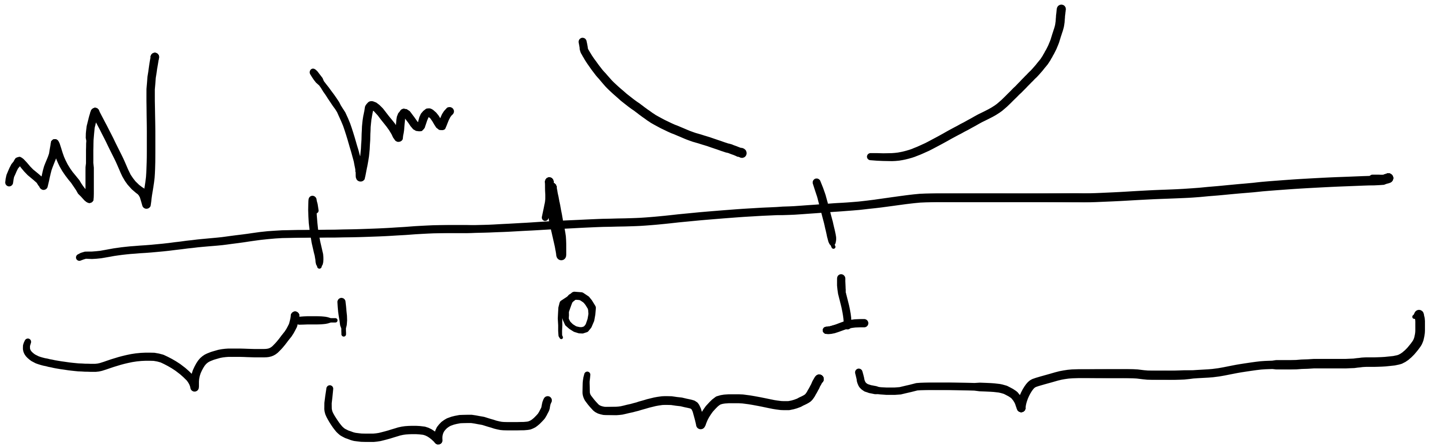
$P_1 = \underbrace{-0,5}\, P_0$

$P_2 = -0,5 P_1 = \underbrace{0.25}\, P_0$

A solução é em zigzag e decrescendo

$\beta^2 - 4\gamma > 0$

ou seja $\beta^2 > 4\gamma$



cresc. em zig zag

decresc em zig-zag

decresc exp

cresc exp

E se $\beta^2 < 4\gamma$

$$R_\pm = \frac{\beta}{2} \pm \frac{1}{2}\sqrt{\underbrace{\beta^2 - 4\gamma}}$$

$$R_\pm = \frac{\beta}{2} \pm \frac{i}{2}\sqrt{4\gamma - \beta^2}$$

$$R_\pm = A + iB$$

$$R_\pm = \underbrace{\frac{\beta}{2}}_{a} \pm \frac{i}{2}\sqrt{4\gamma - \beta^2}$$

$$\rho = \sqrt{\left(\frac{\beta}{2}\right)^2 + \left(\frac{1}{2}\sqrt{4\gamma - \beta^2}\right)^2}$$

$$= \sqrt{\cancel{\frac{\beta^2}{4}} + \frac{1}{4}4\gamma - \cancel{\frac{1}{4}\beta^2}} =$$

$$= \sqrt{\gamma}$$

$$z = \underbrace{a}_{} + i\underbrace{b}_{}$$

$$z = \underbrace{\rho}_{}(\underbrace{\cos\theta}_{} + i\underbrace{\sin\theta}_{})$$

$$\rho = \sqrt{(a)^2 + (b)^2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{b}{a}$$

$$\theta = \arctan\left(\frac{b}{a}\right)$$

$$R_{\pm} = \frac{\beta}{2} \pm \frac{i}{2}\sqrt{4\gamma - \beta^2}$$

$$z = a + ib$$

$$z = \rho(\cos\theta + i\sin\theta)$$

$$\theta = \arctan\left(\frac{b}{a}\right)$$

$$\theta = \arctan\left(\frac{\frac{1}{2}\sqrt{4\gamma - \beta^2}}{\beta/\alpha}\right)$$

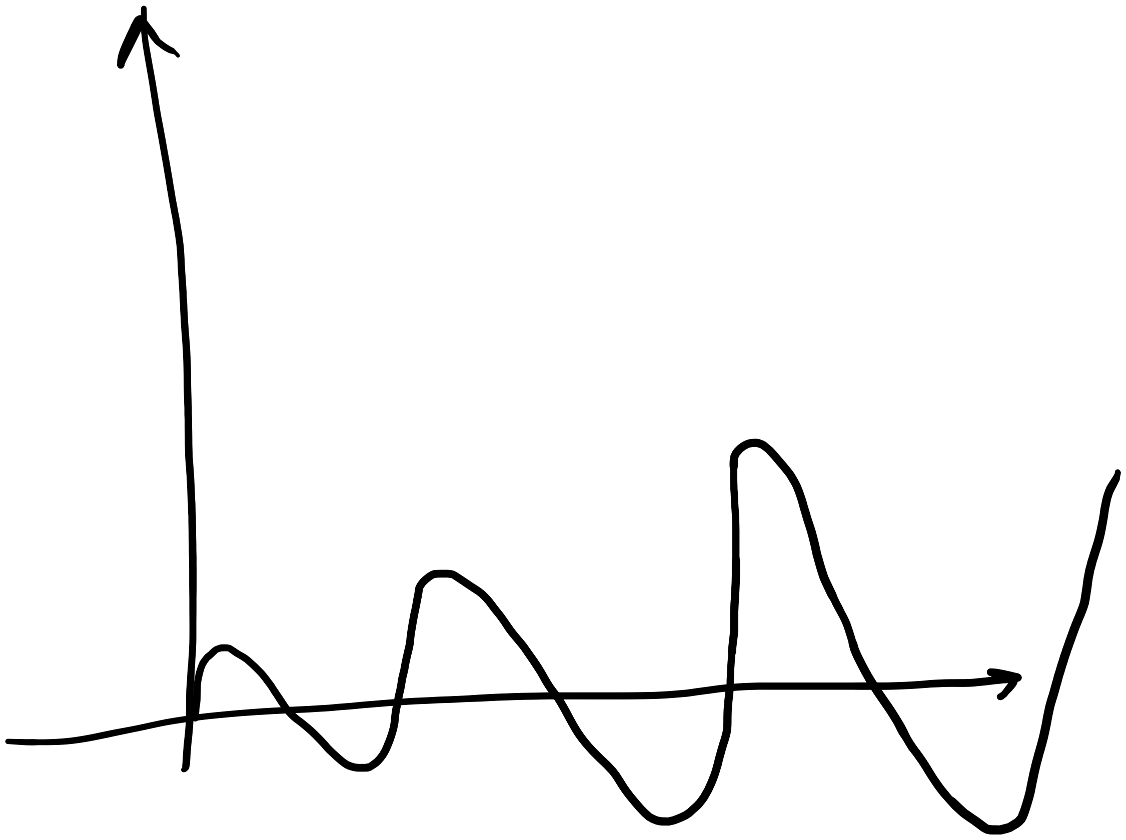
$$R_{\pm} = \rho(\cos\theta \pm i\sin\theta)$$

$$\rho \to \sqrt{\gamma}$$

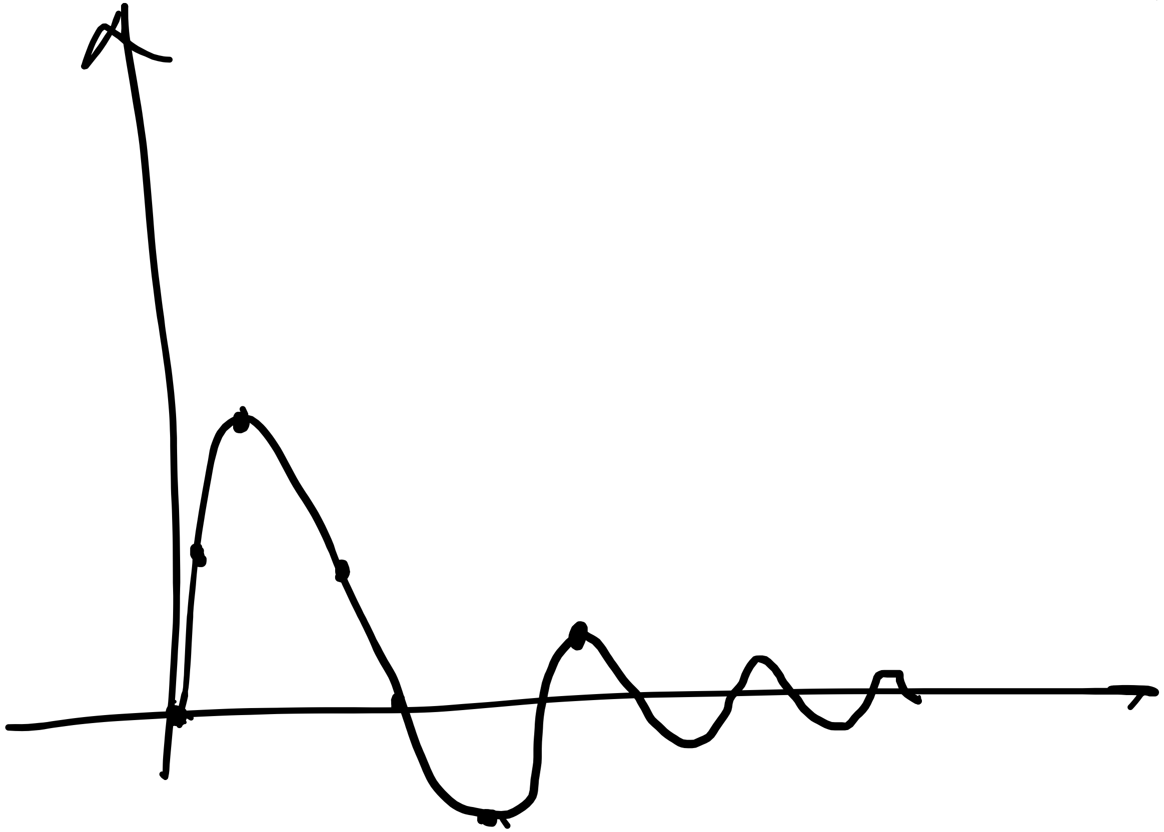
Solução: $P_n = C_+ R_+^n + C_- R_-^n$

$$P_n = A\rho^n \cos\theta + B\rho^n \sin\theta$$

condições iniciais



$\rho > 1$



$0 < \zeta < 1$

zig - zag

$\frac{2\pi}{\theta}$ período das oscilações.