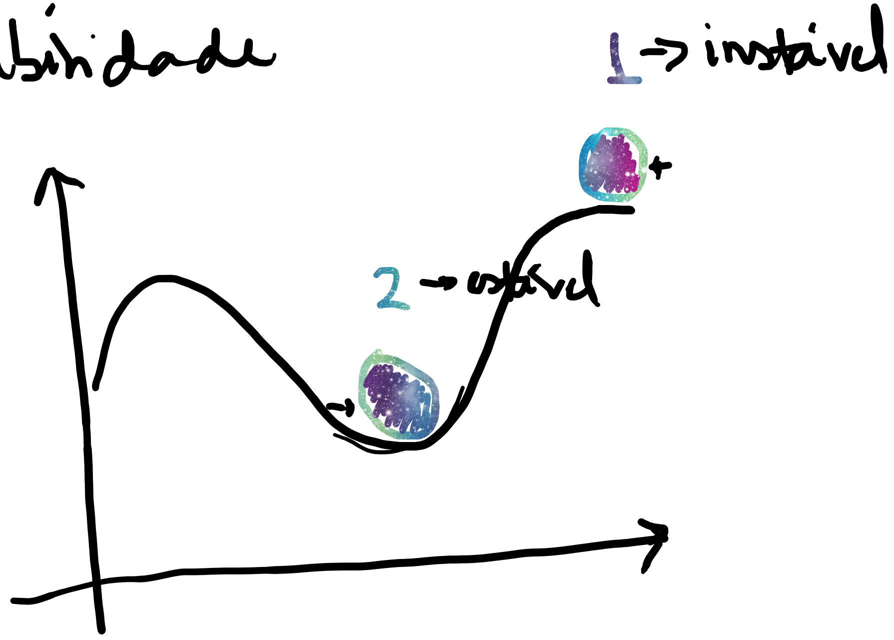
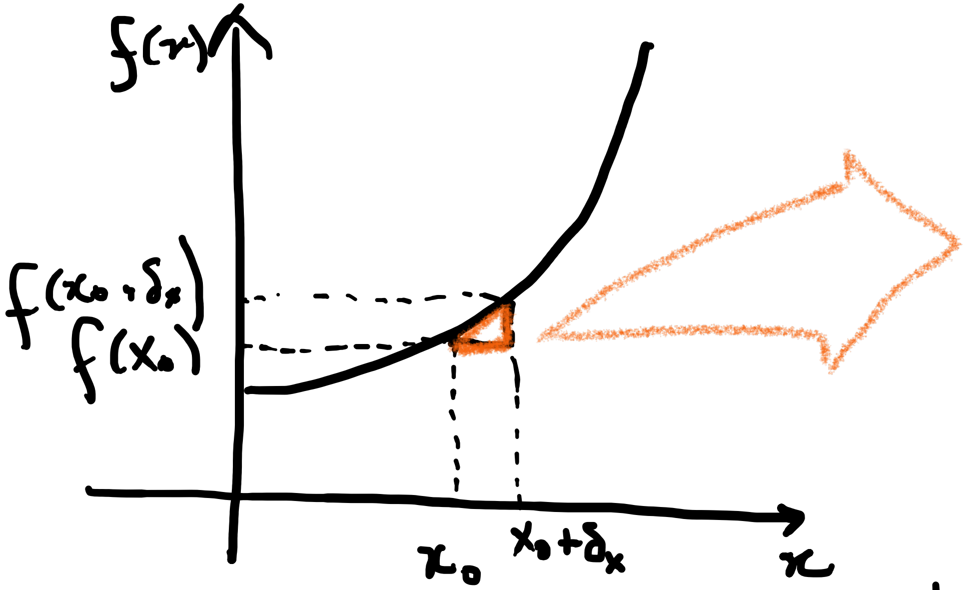
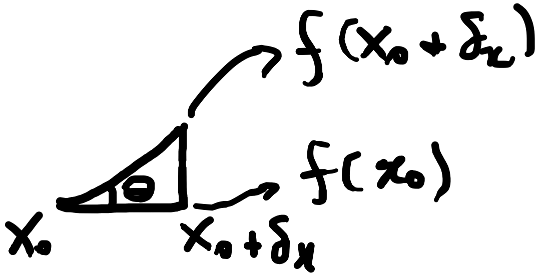
# Estabilidade

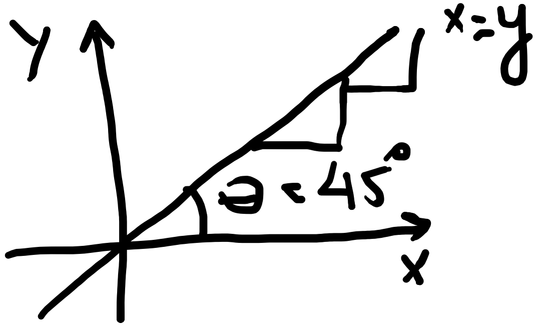



# Derivadas

Derivadas:




$\tan 45° = 1$

$$\tan\theta = \frac{\text{cateto oposto}}{\text{cateto adjacente}}$$

$$\tan\theta = \frac{f(x_0+\delta_x) - f(x_0)}{\cancel{x_0} + \delta_x - \cancel{x_0}} = \frac{f(x_0+\delta_x) - f(x_0)}{\delta_x}$$

$$\lim_{\delta_x \to 0} \frac{f(x_0+\delta_x) - f(x_0)}{\delta_x} = \frac{df}{dx}(x_0)$$

$$\frac{f(x_0+\delta_x)-f(x_0)}{\delta_x} \approx \frac{df}{dx}(x_0)$$

$$f(x_0+\delta_x)-f(x_0) = \frac{df}{dx}(x_0)\,\delta_x$$

$$\boxed{f(x_0+\delta_x) = \frac{df}{dx}(x_0)\,\delta_x + f(x_0)}$$

$\bar{x}$ → Estabilidade de $\bar{x}$?

$$\boxed{x_{n+1} = f(x_n)}$$

$$x_{n+1} = f(\bar{x}) = \bar{x}$$

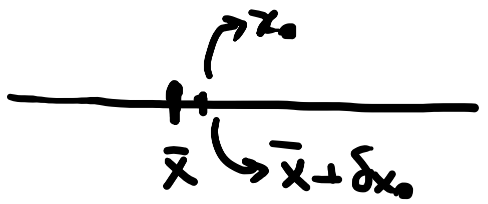
$$\bar{x} + \delta_{x_0} = x_0$$

$$x_1 = f(x_0)$$

$$x_1 = f(\bar{x} + \delta x_0)$$

$x_1$ está perto do $\bar{x}$.

$$x_1 = \bar{x} + \delta x_1$$




$$\bar{x} = f(\bar{x})$$

$$x_1 = \bar{x} + \delta x_1 = \underbrace{f(\bar{x} + \delta x_0)}$$ ★

$$\bar{x} + \delta x_1 = \frac{df}{dx}(\bar{x})\,\delta x_0 + \underbrace{f(\bar{x})}_{\bar{x}}$$

$$\cancel{\bar{x}} + \delta x_1 = \frac{df}{dx}(\bar{x})\,\delta x_0 + \cancel{\bar{x}}$$

pert de $x_1$ ← $\underbrace{\delta x_1} = \boxed{\frac{df}{dx}(\bar{x})}\,\underbrace{\delta x_0}$ → pert de $x_0$

$$\left|\frac{df}{dx}(\bar{x})\right| < 1 \quad \rightarrow |\delta_{x_1}| < |\delta_{x_0}| \rightsquigarrow \text{estável}$$

$$\left|\frac{df}{dx}(\bar{x})\right| > 1 \quad \rightarrow |\delta_{x_1}| > |\delta_{x_0}| \rightsquigarrow \text{instável}$$

Problema da saturação

$$x_{n+1} = \underbrace{\frac{a x_n}{1 + a x_n}}_{f(x)}$$

$$x_{n+1} = x_n = \bar{x}$$

$$\bar{x} = \frac{a\bar{x}}{1 + a\bar{x}}$$

$$\bar{x}(1 + a\bar{x}) = a\bar{x}$$

$$\bar{x}(1 + a\bar{x}) - a\bar{x} = 0$$

$$\bar{x}(1 + a\bar{x} - a) = 0$$

$$\bar{x} = 0$$

$$1 + a\bar{x} - a = 0$$

$$a\bar{x} = a - 1$$

$$\bar{x} = \frac{a-1}{a} = 1 - \frac{1}{a}$$

$$\frac{df}{dx} = \frac{d}{dx}\left(\frac{ax}{1+ax}\right)$$

$$f(x) = \frac{ax}{1+ax} = ax(1+ax)^{-1}$$

Regra do produto

$$\frac{d}{dx} f(x)g(x) =$$

$$\frac{df(x)}{dx} g(x) +$$

$$\frac{dg(x)}{dx} f(x)$$

$$= \frac{d}{dx}\left(\underbrace{ax}\cdot\underbrace{(1+ax)^{-1}}\right)$$

$$= a(1+ax)^{-1} - (1+ax)^{-2} a\, ax$$

$$= \frac{a}{1+ax} - \frac{a^2 x}{(1+ax)^2}$$

$$= \frac{a(1+ax) - a^2 x}{(1+ax)^2} = \frac{a + \cancel{a^2 x} - \cancel{a^2 x}}{(1+ax)^2} = \frac{a}{(1+ax)^2}$$

$$\frac{df}{dx} = \frac{a}{(1+ax)^2}$$

$$\left|\frac{df}{dx}(\bar{x})\right| < 1 \rightarrow \text{estável}$$

$$\left.\frac{df}{dx}\right|_{\bar{x}=0} = \frac{a}{(1+0)^2} = a$$

$$\left|\frac{df}{dx}\right| > 1 \rightarrow \text{instável}$$

$$\bar{x}=0 \begin{cases} a < 1 \rightarrow \text{estável} \\ a > 1 \rightarrow \text{instável} \end{cases}$$

$$\frac{df}{dx} = \frac{a}{(1+ax)^2}$$

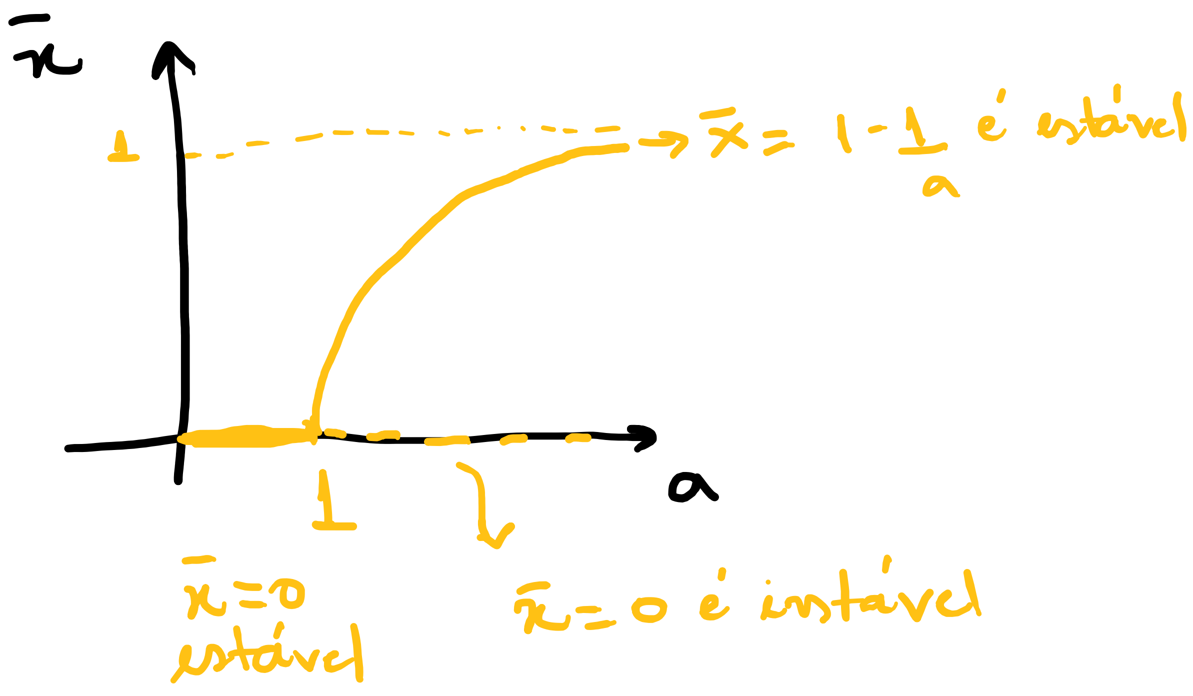
$$\bar{x} = 1 - \frac{1}{a}$$

$$\left.\frac{df}{dx}\right|_{\bar{x}=1-\frac{1}{a}} = \frac{a}{\left(1+a\left(1-\frac{1}{a}\right)\right)^2} = \frac{a}{(\cancel{1}+a-\cancel{1})^2} = \frac{a}{a^2} = \frac{1}{a}$$

$$\bar{x} = 1 - \frac{1}{a} \qquad a > \cancel{1} \qquad a < -1$$

$$\frac{df}{dx} = \frac{1}{a} \Rightarrow \left.\frac{df}{dx}\right|_{\bar{x}} < 1 \text{ sempre que } a > 1$$

$$\bar{x} = 1 - \frac{1}{a} \rightarrow \text{Estável}$$




$$\bar{x} = 0$$

$$\bar{x} = 1 - \frac{1}{a}$$

Desafio:

Estabilidade dos pontos fixos de:

$$x_{n+1} = \underline{R_0 x_n e^{-x_n/k}}$$

fixos

$x_{n+1} = x_n = \bar{x}$ ✓

$\left.\frac{df}{dx}\right|_{\bar{x}}$ ✓

$$f(x) = R_0 x e^{-x/k}$$